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Exploration of the Magic Formula as a Basis for the Modelling of Tyre-Soil Interaction

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Abstract

One of the most important aspects of the theory of cross-country mobility is the modelling of the soil-tyre interaction, i.e. the accurate determination of tyre mechanical and kinematic interaction which takes place in the soil-tyre interface. Due to the drastic increase in the demand for cross-country vehicles today, available multi-body software packages such as ADAMS or DADS are in need of a practical dynamic tyre-soil interaction module. It is almost ridiculous that complex models of a large number of degrees of freedom employ simplifying assumptions such as a rigid soil surface or rigid wheels.

Manufacturers require today a practical tyre-soil model and thereby the augmentation of the commercial multi-body vehicle dynamics via this new "tool-box".

Sofar, different trends can be distinguished in tackling the tyre-soil interface. A first approach is to use the full classical mechanical constitutive relationships and conservation laws. These methods have a general validity but are approximative and not efficient in use. The second approach consists of employing purely empirical ways, yielding accurate answers for a specific range of conditions without however possessing general validity. As a result, there is a need for a general purpose tyre-soil model combining the advantages of general validity and high accuracy.

In contrast to tyre-soil interaction, models describing the interaction between a tyre and a smooth nondeformable surface are well developed. One of the most widely used empirical tyre model is based on the so-called Magic Formula (MF), the development of which started in Delft in the mid-eighties and which can now be regarded as the world standard in modelling of tyre-road interaction. The Magic Formula is capable of producing characteristics that closely match measured curves for the side force and longitudinal force as function of their respective slip

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quantities. This paper explores the exploitation of the Magic Formula as a basis for longitudinal tyre-soil interaction.

After a discussion of the mechanical process of the tyre interacting with soft soil, a description of the Magic Formula is given. Subsequently, this Magic Formula is successfully fitted to peripheral force versus slip curves measured in soft soil. MF- coefficients are determined for various tyre types, with varying soil parameters such as internal friction, cohesion, porosity, moisture content, as well as tyre conditions in terms of inflation pressure and normal load. Clearly, in spite of this variation, this fit merely shows the validity of the Magic Formula for the specific tyre-soil combinations at hand covering a small subset of the possible tyre-soil combinations. For that reason, the relationship between the empirical MF-factors (constants) and soil parameters is studied in a theoretical way by analytical comparison of the Magic Formula with more established empirical models for tyre-soil interaction (particularly those due to Komandi).
As a result, explicit expressions for the Magic Formula coefficients are obtained in terms of soil parameters allowing the application of this pragmatic MF-approach beyond the available experimental soil characteristics. The qualitative validity of these dependencies is discussed. Since the Magic Formula tyre model is available with most leading multi-body packages, this may open the door to a more universal and standard tyre-soil interaction model throughout the world.

1. Introduction and background

During the first decades of this century researchers concentrated on the analytical determination of the rolling resistance. Progress in this field finally began to accelerate in 1954 when M.G. Bekker founded the Land Locomotion Laboratory. Researchers in this laboratory analyzed the soil-tyre interaction in a systematic way. They went beyond rolling resistance and worked on the analytical determination of the tractive force. It was soon realized that the soil undergoes dual loading under a driven wheel. The mass of the vehicle exerts a vertical force which compresses the soil. The soil develops a reaction force whose moment taken to the centre of the wheel has to be balanced by the driving torque.
The driving torque exerts a shear-load underneath the wheel. The shearing action causes relative motion between the tyre and the soil as well as between different soil surfaces underneath. The exact shape and location of these surfaces are very hard --if not impossible-- to determine. The relative motion causes increasing shear deformation and shear stresses. When the resultant reaction forces reaches the magnitude of the active shear force, the tyre rolls along the soil surface. Most researchers agree that sinkage of the tyre and the resulting soil compaction generate the rolling resistance encountered in soft soil. Further energy-loss is caused by the slip, i.e. the relative motions described above and the deflection of the tyre. A search has long been

going on for a general relationship describing this mechanical process. Many simplifying assumptions have been employed because the process is very complex.

- Rolling resistance was associated with the vertical compression of the soil and, hence, with wheel sinkage.

- From the soil-shear diagram, a peripheral force was determined, acting at an undetermined location on the tyre surface. The following factors were considered here: the normal load, the tyre-soil contact surface, soil cohesion and internal friction and the slip which characterizes the relative motions discussed earlier.

Two trends became prominent. Followers of one school tried to describe the process by means

of measurable "natural" parameters and employed empirical factors only to bridge the gap between theory and reality. In these models empiricism is not the main supporting pillar, it merely provides specific connecting elements. These computing methods are approximative, but nobody questions their general validity.

Followers of the second trend chose to bypass natural parameters for the description of the process. They rather employed purely empirical ways. These methods yield accurate answers for a specific range of conditions but they don't possess general validity.

We will summarize the different mathematical methods as found in publications related to cross-country mobility. This will be followed by the description of the Magic Formula which was created for modelling the relationship between the tyre and a hard road-surface. This Magic Formula is well suited for calculation of tyre forces and moments in vehicle simulation studies because of its generality and its well establised link with the major commercially available vehicle system dynamics codes.
The Magic Formula (MF) is fitted here to peripheral force versus slip curves measured in soft soil. We examine the relationships between the empirical factors (constants) of the MF and soil parameters, based on an analytical comparison with empirical models. This endeavour faces difficulties because there are many types of soil and even one particular soil may exhibit different conditions as its compaction and/or moisture content changes. On the other hand, these relationships allow the extended application of existing MF-based commercial software tools for tyre-road interaction to typical deformable surfaces.
An additional problem is caused by the fact that we don't have complete sets of test data which, for example, would cover all soil cohesion values at a given moisture content. We have single curves, rather than a complete family of curves. The main reason for not having complete sets is that it is very hard to prepare or create soils of predetermined structure and condition.

2. The mechanical process between tyre and soil.

For the sake of simplicity we will first examine the case of a rigid tyre rolling on deformable soil, where one may distinguish between towed and driven wheels. There is no active torque applied to the axle of a towed wheel, whereas driven wheels are forced to rotate by the driving torque generated by the engine and transmitted by the power train. Bearing friction will be ignored in both cases.

Towed Wheels.

The situation is sketched in figure 1.

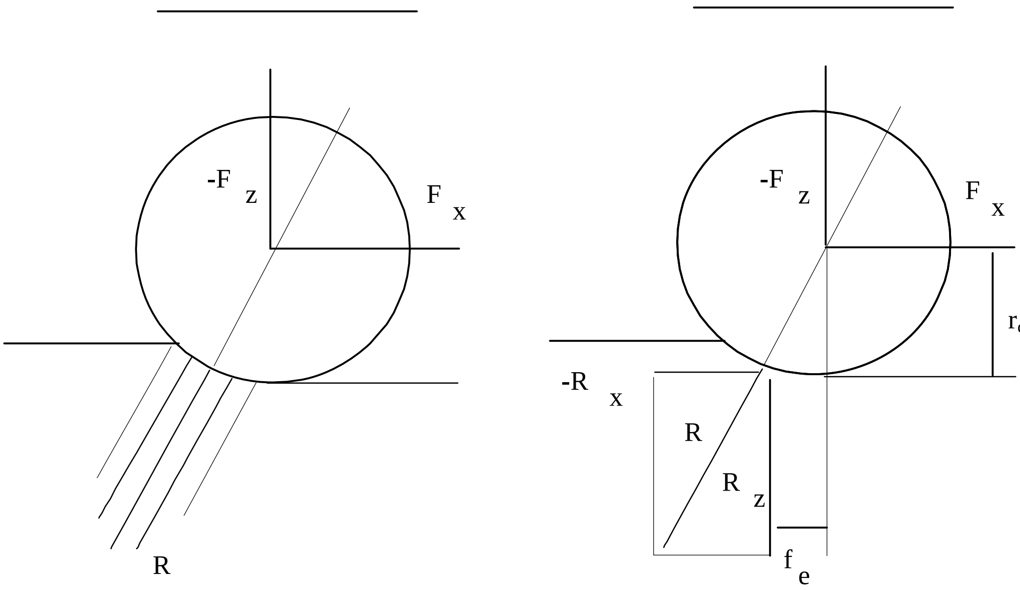

Figure 1.: A Towed wheel

A wheel exerts a force to the soil under the action of the vertical load $-F_z$ and the towing force F_x, acting at the centre of the wheel axle and pointing into a direction parallel to the ground upper surface. This results into a reaction force R between tyre and soil, with components R_z and (rolling resistance) $-R_x$. This latter component is close to being tangential to the tyre perimeter at distance r to the wheel centre with loaded tyre radius r, and at distance f_e ahead of the projection of the wheel centre to the surface plane.

Equilibrium of forces and moments is mathematically expressed as follows:

Horizontal forces: $F_x = - R_x$

Vertical forces: $- F_z = R_z$

Moments: $R_x.r + R_z.f_e = 0$

From these equations one can express the rolling resistance which acts against the towing force

$$R_x = - R_z.f_e/r = F_z.f_e/r$$

demonstrating the fact that the rolling resistance depends on the normal force and the longitudinal wheel slip. The latter depends on the bearing capacity of the soil.

Driven wheels.

Driven wheels are rotated by the torque generated by the engine and transmitted through the drive train. This forced rotation causes the vehicle to move over the soil surface.

Both normal stresses and shear stresses in the contact zone are effected by the driven wheel.

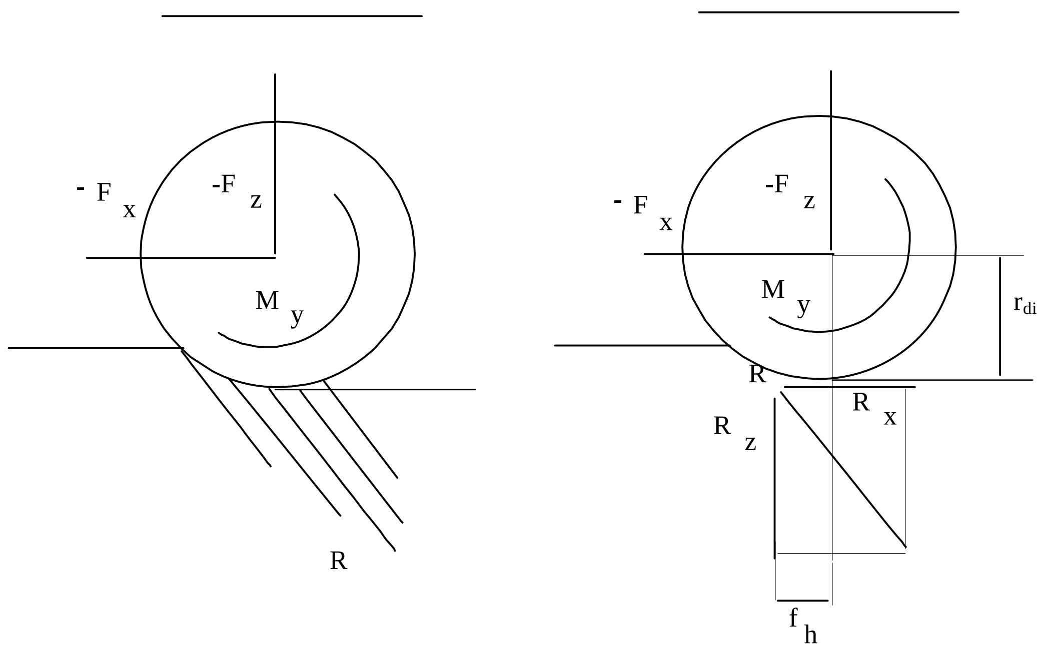

Figure2.: A driven wheel

Driven wheels are acted upon by the following forces and moments (see figure 2):

- part of the vehicle's total mass $-F_z$
- drawbar force $-F_x$, acting at the centerpoint (axle) pointing into a direction parallel to the ground surface, and pushing the wheel backwards (with the wheel moving forwards)
- driving torque M_y.
- reaction force distributed along the soil-tyre interface, whose resultant R can be resolved into a component R_x in the direction of vehicle motion which is approximately at a distance r (loaded tyre radius) to the wheel centre, and the normal force R_z pointing out of the soil. This latter force is located in front of the projection of the wheel centre to the ground surface, its distance to the projection being denoted as f_h.

Equilibrium of forces and moments:

Horizontal forces: $- F_x = R_x$

Vertical forces: $R_z = - F_z$

Moments: $R_z.f_h + R_x.r = M_y$

The force R_x (referred to by Komandi [5] as the driving force) can be calculated as the difference of the maximum circumferential force F_t and rolling resistance F_g of the driven wheel:

$$- F_x = R_x = F_t - F_g$$

with

$$F_t = M_y/r$$

resulting in

$$F_g = R_z.(f_h/r)$$

One should bare in mind that both forces F_t and F_g are fictious forces. In reality only F_x exists, being however unequal to the maximum circumferential force.
The fact that the coefficient of rolling resistance is a function of soil deformation poses further problems. To simplify the problem, camber is neglected for driven wheels, which is well justified especially for slow moving agricultural tractors or any other off-road vehicles which are equipped with low inflation tyres.

3. The analytical method of computation of the driving force.

Shear deformation curves (shear stress versus shear displacement) are usually taken as monotonously increasing curves, asympotically approaching a maximum shear stress for increasing shear displacement. It has been shown that soils may also exhibit "humped" diagrams, i.e. a local maximum stress is observed after which the shear stress decreases to a constant value of residual stress with growing shear displacement. This includes the special situation of the soil being rigid, i.e. in case of a road surface.
The character of these curves is a function of the physical state of the soil, with soil compaction and moisture content as primary influencing factors. Several mathematical representations are discussed by Wong and Preston-Thomas in [9].

Bekker accepted *humped* diagrams as the general form for soil deformation representation. He proposed the following expression for their mathematical representation:

$$t = (c + p.m).[exp\{(-K_2+Ö(K_2^2-1)).K_1.u\} - exp\{(-K_2-Ö(K_2^2-1)).K_1.u\}]/Y$$

with cohesion c, internal friction m, contact pressure p, horizontal soil displacement u and empirical factors K_1 and K_2. The denominator Y is the maximum of the expression in the numerator. Determining the constants in Bekker's equation (applicable to tracked vehicles) is complicated and one can easily commit a large error in reading off their values on a log-log plot. Wong and Preston-Thomas have suggested a simpler form in [9].
For soil not exhibiting a "hump", Janosi and Hanamoto proposed the following expression for *asymptotic* shear diagrams, being widely used in practice because of its simplicity:

$$t = (c + p.m).[1 - e^{-b.u}]$$

for empirical factor b. Assuming that the contact surface is constant as well as the slip in the contact area, the driving force follows from:

$$-F_x = A.(c + p.m).[1 - (1 - e^{-bL.k})/(bLk)]$$

for dimensionless longitudinal slip k (fractional reduction in forward speed), contactlength L and ground contact area A (applicable to tracked vehicles). Komandi extended the range of application of this equation by obtaining b from traction versus slip curves measured in the field using actual vehicles. In this way, the actual mechanical process happening underneath the vehicle was taken into consideration.

The conditions of soil-tyre interaction are much more complex than for tracked running gears. Shear takes place under the deformed surface of the tyre, that is we are not dealing with a shear plane. The tractive force and the rolling resistance affects the rear axle load, and thus the shape and area of the contact surface change. Consequently, the specific soil pressure is not constant. According to test results by Komandi [5] the contact surface of a tyre depends on the construction of a tyre, but it also depends on the normal load and from the inflation pressure. According to Komandi, the main influencing factor on the tractive force is the active portion of the tyre-soil interface surface. The effect of the shear stress is only secondary. Therefore he proposed the following equation:

$$-F_x = t \, . \, \check{n} \, dA$$
$$A_{sh}$$

instead of the original one which is as follows:

$$L$$
$$-F_x = 2b \, . \, \check{n} \, tdx$$
$$0$$

for half-width b of the contact surface and A_{sh} being the part of A which slides on the ground (shear surface). In Komandi's equation shear stress t is constant, which is diametrically opposed to the teachings of classical mechanics. One can however regard this as neglecting the range from zero to t_{max} = c + p.m. One should note that in reality this section represents a very small shear displacement because of the shear curve reaches its maximum very "quickly". Once t_{max} is reached it remains constant troughout the whole range. Soehne also found that the surface along which force transfer occurs depends on the normal load, the slip, the type of soil and its condition.

A serious drawback of Komandi's equation is that the active part of the soil-tyre interface cannot be established experimentally so it can only be determined indirectly. Therefore his method can only be checked indirectly as well. This surface can only be approximated by means of empirical factors. Another difficulty stems from the fact that cohesion and friction are not constant for a given soil but vary with slip even for the same tyre and soil.

It is clear that the establishment of a soil-tyre model has not been possible up to now. The question is whether we should approach the problem purely empirically or should strive for a general method using natural parameters.

4. The Magic Formula tyre model.

In contrast to tyre-soil interaction, models describing the interaction between a tyre and a smooth nondeformable surface are well developed. One of the most widely used empirical tyre model is based on the so-called Magic Formula, the development of which started in the mid-eighties. Various improvements have been established since then resulting in the newest version denoted as Delft-Tyre 96. The basis for the Magic Formula (or MF for short) is the steady state

tyre behaviour. This has been extended in the Delft-Tyre version to transient tyre behaviour as well. We refer to [7] for a full description of the model.

The first Magic Formula was presented in 1987 and extended in 1989 describing pure and combined slip, based on a physical background, where tyre relaxation lengths were introduced in order to have a first order approximation of the dynamic tyre behaviour. The most recent Magic Formula Delft-Tyre version has been further improved, with respect to the following aspects:

- A reduced complexity of the force calculations and combined slip calculations, resulting in a much higher calculation speed
- a self aligning torque is based on the pneumatic trail and in that way made dependent on the side force

- low speed and tyre loading variations to tyre lift situations have been included
- parameters used in the formulae are dimensionless, improving manipulations with tyre characteristics and parameter calculations
- scaling factors are introduced for vehicle-tyre optimization purposes.

Beépiteni az ábrát

Figure 3.: Indication of Magic Formula Parameteres

The basic formula reads:

$$y = D \sin [C \arctan \{Bx - E(Bx - \arctan Bx)\}]$$

with

$$Y(X) = y(x) + S_v$$
$$x = X + S_h$$

where (see figure 3):

Y: side force F_y or longitudinal force F_x
X: slip angle a or longitudinal slip k

and coefficients

B: stiffness factor
C: shape factor (= 2.arcsin(y_a/D)/p
D: peak value

E: curvature factor (=[B.x_m-tan(p/(2C))]/[B.x_m-tan(B.x_m)]

Two shifts S_h, S_v have been introduced to represent off-sets of the curves due to tyre irregularities (ply-steer, conicity, etc.). The product BCD represents the slope near the origin (e.g. cornering stiffness). Each of these coefficients depends further on vertical load F_z and camber angle g, see [7].

An advantage of the above basic formula (w.r.t. the expressions due to Janosi, Komandi, etc, see section 5) is that it can be applied both for curves with a local maximum such as for a rigid surface and for certain clayey surfaces, see [9], and for curves which are monotonously increasing such as for a sandy surface. A second advantage is that the basic Magic Formulae can be applied to calculate both the lateral and longitudinal forces and aligning torque (through the pneumatic trail) acting on a tyre under pure and combined slip conditions, from the longitudinal and lateral slip (k, a), wheel camber (g), and the vertical force through the dependence of B, C, D,.... on these inputs.

It is known (see for example [7]) that the Magic Formula is capable of producing characteristics that closely match measured curves for the side force F_y and longitudinal force F_x as function of their respective slip quantities a and k, defined as follows:

$$\tan(a) = -V_{sy}/V_x$$

$$k = -V_{sx}/V_x = 1 - w.r_e/V_x$$

with V_{sx} and V_{sy} the components of the slip speed and V_x the forward speed, w the angular speed of rolling and r_e the effective rolling radius. The aligning torque M_z can be obtained from side force F_y, pneumatic trail and residual torque. This will not be considered in this paper and we refer to [7] for further details. The Magic Formula is the most widely used tyre model for application on a rigid road surface. Recent applications of the Magic Formula can be found in the development of active in-plane control strategies such as DYM (Direct Yaw Moment control) with the objective to keep the vehicle in a stable course under extreme low-friction conditions.

In order to provide the engineer nowadays with a complete tool for analysis of tyre behaviour for vehicle dynamics purposes, the **DELFT-TYRE** productline has been developed around the latest Magic Formula. DELFT-TYRE is a design and analysis toolbox which supports testing and modelling of tyre behaviour. It consists of four different modules:

- DELFT-TYRE Test Trailer
- MF-TYRE
- MF-DATASETS
- MF-TOOL

The **DELFT-TYRE Test Trailer** is a tyre measuring device, allowing the horizontal performance characteristics of passenger cars and motorcycles to be measured under a range of conditions at any locations.
MF-TYRE is a subroutine for modelling and simulation of tyre behaviour.
High quality tyre data (**MF-DATASETS**) is supplied to fit with this model.
MF-TOOL is a module for database management, visualisation of tyre characteristic and preparation of user defined datasets with the aid of a graphical user interface.
MF-TOOL can be extended to **MF-TOOL$^+$**, including the fitsoftware MF-FIT to build one's own tyre characteristic from available tyre measurement data. A simplified version of MF-FIT has been used for the determination of MF-parameters from experimental tyre-soil interaction data, as reported in section 4.

5. Magic Formula fit of some realistic experimental data

The Magic Formula has been applied to soil-tyre interaction data. For that purpose, the experimental data due to Steinkampf [8] has been used. As explained in section 2, the maximum circumferential force (driving force) F_t is composed of the drawbar pull force and the rolling resistance, where the latter behaves approximately as a linear function of the longitudinal slip whereas the former force follows approximately the Janosi's expression.
In this section, experimental data and their MF-fits are mutually compared for this last force R_x. Two different tyre types have been considered:

(i) Continental farmer tyre, size 16.9-30
(ii) Continental farmer tyre, size 18.4-34

on a soil-type which can be described as taft sandy.
Different measurement for various tyres of the same type and size have been considered, with varying soil parameters such as internal friction, cohesion, porosity, moisture content, and tyre conditions in terms of inflation pressure and normal load.
The driving speed for all tests was 3.4 km/h.
The testconditions are summarized in table 1 and 2 below.

Test	Normal load [kN]	Pressure [bar]	Porosit y	Moisture content [%]	Cohesion [kPa]	Internal friction
1	14.32	1.1	43.05	11.3	12.26	0.690
2	10.40	1.1	39.95	11.9	10.24	0.898

3	10.40	1.4	38.95	11.9	12.34	0.896
4	13.34	1.4	37.60	10.5	9.85	0.782
5	16.29	1.4	37.60	11.9	10.95	0.849
6	16.29	1.4	38.95	11.9	9.26	0.781

Table 1: Testconditions for agricultural tyre size 18.4-34

Test	Normal load [kN]	Pressure [bar]	Porosit y	Moisture content [%]	Cohesion [kPa]	Internal friction
7	15.82	1.1	37.95	11.9	10.21	1.12
8	15.82	1.4	37.95	11.9	10.21	1.12
9	15.82	1.1	38.05	14.0	11.21	1.02
10	9.94	1.1	38.95	11.9	10.55	0.80
11	9.94	1.4	38.95	11.9	10.55	0.80
12	12.88	1.4	39.65	11.9	11.55	1.90
13	9.84	1.4	39.65	11.9	10.26	0.98

Table 2: Testconditions for agricultural tyre size 16.9-30

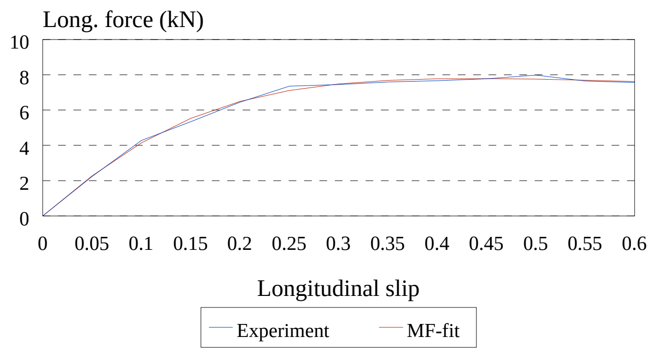

Figure 4.: MF-fit, tyre size 18.4-34

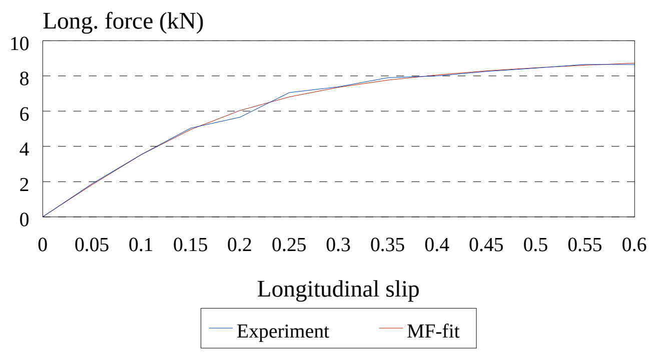

Figure 5.: MF-fit, tyre size 16.9-30

For each of these tyres, the MF-coefficients B, C, D, E, S_h and S_v have been determined using a simplified version of the MF-FIT program, referred to in the preceding section. Typical results are shown in figures 4 and 5, related to test number 5 and 9, respectively.
As one observes, a good correspondence between tests and MF-model is obtained for both tyre types.
The fitresults regarding B, C, D and E for all tyres are included in table 3.

test >	1	2	3	4	5	6	7	8	9	10	11	12	13

B	4.19	5.03	5.26	5.55	3.41	4.97	6.25	6.41	4.67	5.77	6.00	4.84	4.66
C	0.80	0.47	0.33	0.40	1.52	0.41	0.31	0.44	0.47	0.43	0.36	0.45	0.50
D	15.1	13.8	20.6	16.7	9.6	18.4	17.3	11.2	16.3	13.5	14.6	14.8	16.9
E	-0.91	-0.24	-2.46	-2.45	-0.09	0.60	-2.50	-1.22	-1.51	-0.98	-0.24	-1.41	0.54

Table 3: MF-coefficients

6. The dependence of MF characteristics on soil parameters

In this section, the objective is to interpret the Magic Formula description of the longitudinal force (e.g. driving force) in terms of soil characteristics en wheelload variation in a *qualitative* sense. Starting point will be the Komandi expression for the driving force F_x, depending on t_{max} = c + p.m (with cohesion c, internal friction m taken constant for simplicity) and the shear surface A_{sh} depending on longitudinal slip in an exponential way. This expression will allow explicit expressions of the MF-coefficients (stiffness-factor B, peak-factor D, shape factor C) in terms of wheelload, cohesion, internal friction, etc. that can be used as "Ansatz" for future improved pragmatic expressions for B, C and D in terms of tyre load, longitudinal slip and a finite number of soil parameters.
The MF-formula (with shifts not taken into account) and Komandi's expression are compared by equating the value at maximum slip, and the slopes with respect to the slip at both zero and maximum slip.

The Komandi expression can be written as:

$$-F_x = t_{max} \,.\, \text{ň}\; dA$$
$$A_{sh}$$

where A_{sh} can be expressed as (see [5]):

$$A_{sh} = A.[1 - (1 - k).exp(-K_1.L^d.k)]$$

with contact length L, total contact area A depending on tyre load, inflation pressure p_i, tyre geometry:

$$A = F_z^q.A_0(p_i, \text{tyre width/diameter})$$

for some empirical factor q (in [5] taken as 0.7).
The value of d varies between 0.5 and 0.8, as indicated below:

d = 0.5	sand, wheat stubble
d = 0.6	wheat stubble loam soil
d = 0.8	bare loose sand

Clearly, the contact length (as well as the contact area) depends on the tyre load. Let's try to eliminate L. The following expression is due to Bekker, based on the assumption of a circular shaped crosssection of the tyre-soil interface:

$$L = \text{Ö}(1 - w/d).\text{Ö}(d.w) \sim \text{Ö}w \text{ for small } w/d$$

with soil indentation w and tyre diameter d. This expression neglects the tyre deflection and is known to underestimate the contactlength L. However, it serves our purpose for identifying a pragmatic relationship between L and F_z (i.e. to eliminate L).
Use Bekker's well known expression for the contactpressure p:

$$p = p_0.w^n$$

with n varying:

n = 0.3	sandy loam
n = 0.5	clayey
n = 1.1	dry sand
n = 1.5	snow

As a consequence:

$$F_z \sim L.w^n \sim w^{n+''}$$

i.e.

$$L \sim F_z^{1/(2n + 1)}$$

and

$$L^d \sim F_z^e;\ e = d/(2n + 1).$$

With A approximated by L.b, these expressions are comfirmed by Komandi's results yielding L ~ F_z^q with q in the same order of magnitude as the maximum 1/(2n + 1) values. Assuming a well distributed contact pressure within the contact area and baring in mind that we are merely interested in finding similarity expressions for the Magic Formula coefficients (expressions with the correct qualitative variation in the soil parameters), the following expression for the driving force can be derived:

$$-F_x = (c.A_0.F_z^q + m.F_z).[1 - (1 - k).exp(-C_1.F_z^e.k)]$$

for some empirical coefficient C_1, yielding the results in table 4 below.

$-F_x(0;F_z)$	0
$-F_x(1;F_z)$	$[c.A_0.F_z^q + m.F_z]$
$-F_{x,k}(0;F_z)$	$[c.A_0.F_z^q + m.F_z].[1 + C_1.F_z^e]$
$-F_{x,k}(1;F_z)$	$[c.A_0.F_z^q + m.F_z]. exp(-C_1.F_z^e)$

Table 4 : Driving force characteristics according to Komandi

We refer to section 4 for the Magic Formula:

$$F_x(k;F_z) = D.sin[C.arctan(B.k)]$$

with B, C, D depending on F_z, and neglecting (for simplicity) the curvature factor E as well as the vertical and horizontal shifts.
The following results are easily obtained:

$F_x(0;F_z)$	0
$F_x(1;F_z)$	D.sin[C.arctan B]
$F_{x,k}(0;F_z)$	B.C.D.
$F_{x,k}(1;F_z)$	$B.C.D.cos[C.arctan B]/(1 + B^2)$

Table 5 : Driving force characteristics according to Magic Formula

Setting the expressions equal as given in the tables above results in a set of equations for B, C and D. More specific, we will set the Komandi (soil-tyre) expression and the Magic Formula expression equal at both zero and maximum slip (i.e. k=0 and k=1). In addition, we set the slopes equal at both zero and maximum slip.
Define first:

$$G := c.A_0.F_z^q + m.F_z$$

$$H := 1 + C_1.F_z^e$$

The following equations are found:

$$B.C.D = G.H$$

$$D.\sin[C.\arctan B] = G$$

$$B.C.D.\cos[C.\arctan B]/(1 + B^2) = G.\exp(-C_1.F_z^e)$$

From the numerical values for B and C as derived in section 5, the following approximations are found to be applicable:

/1/ sin[C.arctan B] » C.arctan B

/2/ arctan(B) » p/2 - 1/B

resulting in the explicit expression for B, C and D:

$$B = p.H/4 + [p^2.H^2/16 - H]^{1/2}$$

$$C = \arccos[(1 + B^2).\exp(-C_1.F_z^e)/H]/\arctan(B)$$

$$D = G.H/(B.C)$$

6. Concluding remarks.

Some first observations can be made from the above explicit expressions for the Magic Formula coefficients B, C and D. The shape factor C, determining the global qualitative behaviour of the driving force vs. slip (i.e. wether it is monotoneous or shows a "hump") doesn't show a dependence on cohesion or internal friction. C depends on the tyre load F_z and empirical parameters C_1 and e through $C_1.F_z^e$ describing the increase of the sliding part of the contact area

at low slip. The longitudinal stiffness BCD and maximum driving force D both do depend on cohesion and internal friction in a linear way.
The variation of the coefficients in tyre load F_z is shown in figure 6, for C_1=0.22...., e=0.25...., q=0.7...., A_0=0.0002 ....., c=8 kN/m^2, m=0.7

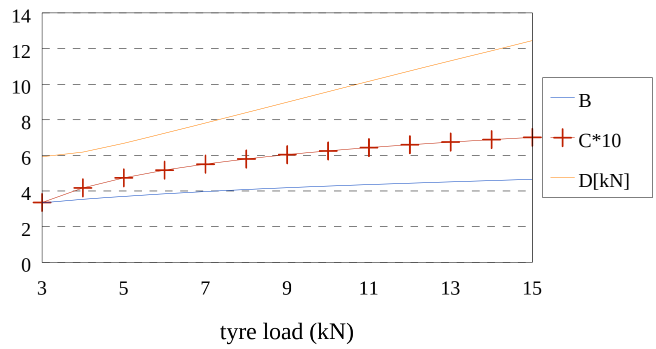

Figure 6.: MF-Coefficients B, C and D.

It is observed that all coefficients increase with F_z, with the behaviour slightly nonlinear for small values of tyre load. The peak value D shows an almost linear behaviour, to a large extent determined by cohesion and internal friction.

Summarizing, it has been demonstrated that the Magic Formula similarity approach is well suited to describe tyre-soil interaction in longitudinal direction. It can describe both monotoneous curves and "humped" diagrams as illustrated in figure 7.

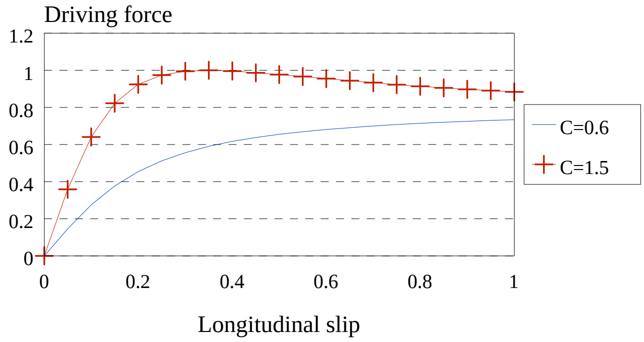

Figure 7.: Two typical MF-curves (B=5, D=1).

The simplified MF-model as used in this paper restricts the model parameters to only three, each of which can be linked to empirical parameters relating to the soil itself and the development of shear surface as depending on slip and load. The dependencies should be further explored in the future to obtain a stronger basis for the Magic Formula model. Since the Magic Formula tyre model is available with most leading multi-body packages, this may open the door to a more universal and standard tyre-soil interaction model throughout the world.

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